



## Art of Science Community...

**When: February 18th, 5-7pm**

**Where: Perry Field House, BGSU**

**Who: Emily Sautter**

**NW Ohio Wind Energy Research**

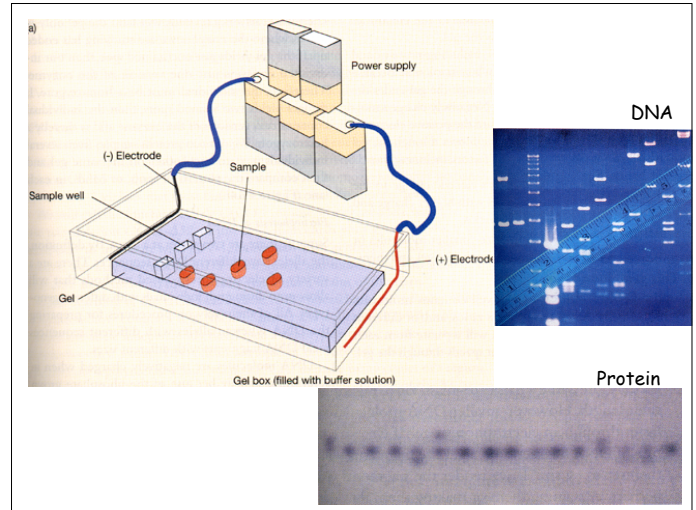
- Qualify for Summer Research!
- Career and Networking Opportunities!
- Door Prizes!



Contact **Liz Ross**, Program Manager, if you have any questions...

419-372-4238  
[setgo.bgsu.occ@gmail.com](mailto:setgo.bgsu.occ@gmail.com)  
 304 Life Sciences, BGSU

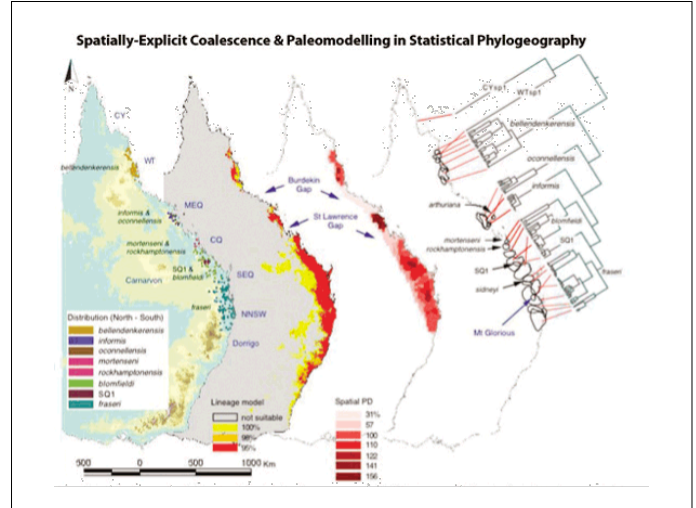
1



4

## Population genetics I: Hardy-Weinberg

2



5



3

"The science of population genetics is the automechanics of evolutionary biology."

Richard Lewontin, 2000

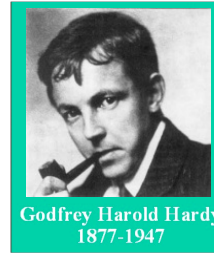
6

A model for gene frequencies in the absence of evolution.....

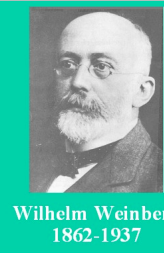
Does the gene pool predict the distribution of genotypes within a population?

7

1908: Godfrey H. Hardy (mathematician)  
Wilhelm Weinberg (physician)  
Sergei Chetverikov (geneticist)



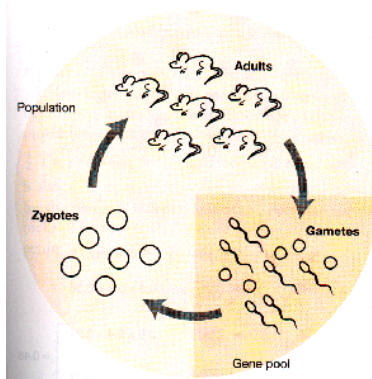
Godfrey Harold Hardy  
1877-1947



Wilhelm Weinberg  
1862-1937

Hardy-Weinberg (HWP) principle  
expectation/equilibrium/equation

10



Allele Frequency: the proportion of all alleles of the gene that are of the prescribed type

8



AA or Aa  
aa → multi



11

Information needed to describe the genetic composition of a population?

- # alleles at the locus
- freq of alleles
- freq of genotypes

	A1A1	A1A2	A2A2
Population 1	50	0	50
Population 2	25	50	25

9

Assume a gene with 2 alleles.....A & a

Allele Frequency:

Freq of A = p (range from 0-1)

Freq of a = q

Sperm

Genotype frequency:

For random mating with

2 alleles at frequencies

p & q

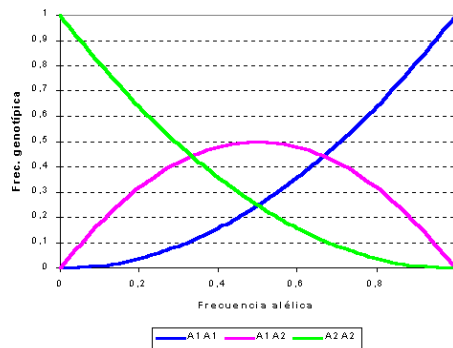
Egg

Allele Frequency	A p	a q
A p	AA p <sup>2</sup>	Aa pq
a q	aA pq	aa q <sup>2</sup>

12

Therefore the HW expectation is.....

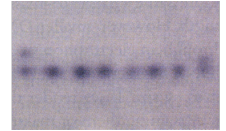
$$p^2 + 2pq + q^2 = 1 \quad (\text{Binomial law})$$



13

#### A numerical example....

Sample a squirrel population (N= 73)  
MDH locus >> 2 alleles, Medium & Fast



3 genotypes possible

MM 20  
MF & FM 36  
FF 17  
Total N = 73 animals



16

#### Assumptions.....

- No selection
- No mutation
- No migration
- No random events
- Pop. infinitely large, stable, equal sex ratio
- Mating within population at random (panmixia)  
i.e. No evolution!  
.....an ideal population

14

3 genotypes possible

MM 20  
MF & FM 36  
FF 17  
Total N = 73 animals

Calculate allele frequencies.....

$$M = 76 \quad P_m = 76/146 = 0.5205$$

$$F = 70 \quad P_f = 70/146 = 1 - P_m = 0.4795$$

Total alleles 2N = 146

17

#### Significance of HW....

$$p^2 + 2pq + q^2 = 1$$

- Allele frequencies predict genotype frequencies
- At equilibrium, allele & genotype frequencies do not change from generation to generation
- If population displaced from equilibrium, return to equilibrium occurs in one generation

15

Do these data conform to HW expectations?

	MM	MF	FF
Expected proportions	$0.5205^2 = .2709$	$2(0.5205 \times 0.4795) = .4992$	$0.4795^2 = .2299$
Expected number	$.2709 \times 73 = 19.776$	$.4992 \times 73 = 36.4426$	$.2299 \times 73 = 16.783$
Observed number	20	36	17

18

Do these data conform to HW expectations?  
(Chi-square Test)

$H_0$ : No deviation from expected proportions

$$X^2 = \sum \frac{(\text{Obs} - \text{Exp})^2}{\text{Exp}}$$

$$= \frac{(20 - 19.776)^2}{19.776} + \frac{(36 - 36.442)^2}{36.442} + \frac{(17 - 16.773)^2}{16.773}$$

$$= 0.00251 + 0.005427 + 0.00277$$

$$X^2 = 0.011$$

19

Ideal populations are entirely predictable:

No...

- selection
- mutation
- migration
- random events
- infinitely large, stable, equal sex ratio
- random mating (panmixia)



22

To check significance.....

$df = \# \text{ of classes} - 1 - \# \text{ parameters calculated from data}$   
 $= 3 - 1 - 1 = 1$

Critical  $X^2$  at  $\alpha 0.05 = 3.1415$

Since  $X^2 < X^2_{.05} = 3.1415$  we fail to reject the  $H_0$   
 i.e. the data conform to HW

If  $X^2 > X^2_{.05}$ .....

then we reject  $H_0$ , conclusion is that the population is not in HW and (at least) one of the assumptions of HW is invalid

20

$$\begin{aligned} \text{freq.}(A1A1 \text{ in zygotes}) &= p^2 & (1) \\ \text{freq.}(A1A2 \text{ in zygotes}) &= 2pq & (2) \\ \text{freq.}(A2A2 \text{ in zygotes}) &= q^2 & (3) \end{aligned}$$

- If all assumptions are true, then equations 1-3 must be true.
- If genotypes are in HW proportions, one (or more) of assumptions may still be violated.
- If genotypes are not in Hardy-Weinberg proportions, one or more of assumptions must be false.

23

2 additional wrinkles in practice....

For loci with more than 2 alleles:

Expand the binomial e.g. for 3 alleles

$$p + q + r = 1$$

$$p^2 + q^2 + r^2 + 2pq + 2pr + 2qr = 1$$

For many loci:

If assume all loci are independent of one another then you can repeat this procedure for each one, add all the  $df$  &  $X^2$  values, and check significance over all loci

21

If population is not in HW  $\Rightarrow$  some assumption has been violated.....

- Random chance/error  
 e.g. inappropriate selection of field site, sample not large enough
- Wahlund effect
  - Sample equally across populations in HW  
 e.g. in aquatic thermocline, or behavioral subdivision
  - decrease hets & increase homozygotes
- Selection
  - decreases hets & increases homozygotes
  - greater likelihood of shared deleterious alleles
- Inbreeding - Breed with others genotypically more like themselves  
 e.g. positive assortative mating, small population sizes

24

### Problems

1. In humans, the *COL1A1* locus codes for certain collagen protein found in bone. The normal allele at this locus is denoted with *S*. A recessive allele *s* is associated with reduced bone mineral density and increased fractures in both *Ss* and *ss* women. A recent study of 1,778 women showed that 1,194 were *SS*, 526 were *Ss*, and 58 were *ss* (Uitterlinden et al 1998).

- (a) Are these two alleles in Hardy-Weinberg equilibrium in this population?
- (b) How do you know?
- (c) What information would you need to determine whether the alleles will be in Hardy-Weinberg equilibrium in the next generation?

25

2. In the peppered moth (*Biston betularia*), black individuals may be either homozygous ( $A_1A_1$ ) or heterozygous ( $A_1A_2$ ), whereas pale gray moths are homozygous ( $A_2A_2$ ). Suppose that in a sample of 250 moths from one locality, 108 are black and 142 are gray.

- (a) Which allele is dominant?
- (b) Assuming that the locus is in Hardy-Weinberg equilibrium, what are the allele frequencies?
- (c) Under this assumption, what proportion of the sample is heterozygous?
- (d) What is the number of heterozygotes?
- (e) Under the same assumption what proportion of black moths is heterozygous?
- (f) Why is it necessary to assume Hardy-Weinberg genotype frequencies in order to answer parts b-d?

26